Quadratics – Completing the Square

When you are asked to complete the square, they want you to go from $f(x) = ax^2 + bx + c$ to $f(x) = a(x-h)^2 + k$ where (h, k) is the vertex.

Shortcut – use if only the final answer is needed

① Note that "a" is the same for both.

② Find the vertex, (h, k): $h = \frac{-b}{2a}$, $k = f(h)$

③ Substitute a, h, k into $f(x) = a(x-h)^2 + k$

and you are done!

The long, tedious method when you must show all steps:

Example: Complete the square to convert $f(x) = 2x^2 - 12x + 14$ to the vertex form.

① Put parentheses around the quadratic and linear terms, then factor "a" out of the parentheses:

$$f(x) = 2x^2 - 12x + 14$$

$$f(x) = 2\left(x^2 - \frac{12}{2}x\right) + 14$$

$$f(x) = 2(x^2 - 6x) + 14$$

② Take $\frac{1}{2}$ of the linear coefficient $\quad \frac{1}{2}(-6) = -3$

③ Square it $\quad (-3)^2 = +9$

④ Add it inside the parentheses, but to keep everything the same, subtract it outside of the parentheses.

$$f(x) = 2(x^2 - 6x + 9) + 14 - 2(9)$$

because the 2 multiplies everything inside, including the 9 ... → you need to multiply the 9 by the 2 before subtracting.

$$f(x) = 2(x^2 - 6x + 9) + 14 - 18$$

← This part is now a perfect square

$$f(x) = 2(x-3)(x-3) - 4$$

$$f(x) = 2(x-3)^2 - 4$$

vertex form, with a vertex of $(3, -4)$

Double check with shortcut:

$h = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$

$k = 2(3)^2 - 12(3) + 14 = -4$

so $(h, k) = (3, -4)$

and $a = 2$

substitute:

$f(x) = 2(x-3)^2 - 4$

same result, less work!