

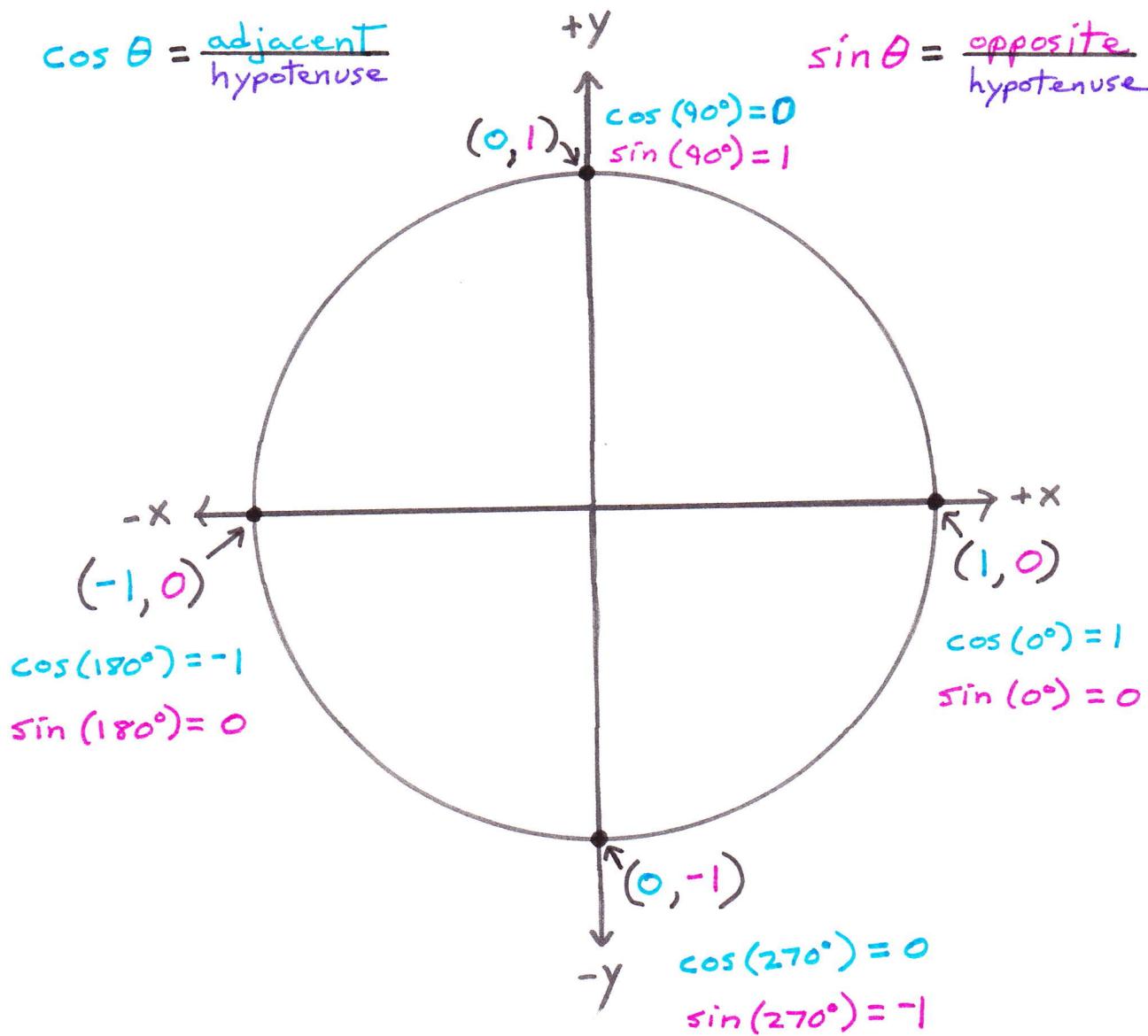
# The Unit Circle

( $x, y$ ) coordinates of a point on the unit circle are

$$(\cos \theta, \sin \theta)$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$



Additional Information:

$$\text{tangent: } \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin \theta}{\cos \theta}$$

# The Unit Circle

for angles that are multiples  
of  $45^\circ$

To find  $a$  &  $b$ , use the  
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

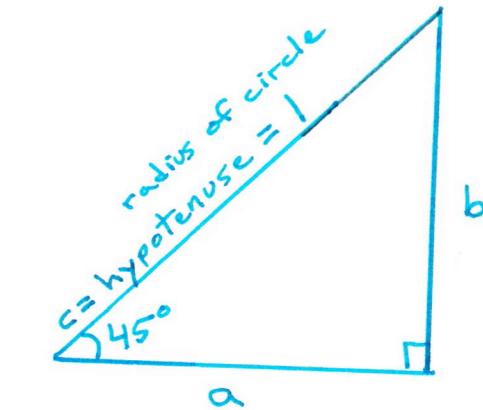
but  $c=1$  so  $c^2=1$

and  $a=b$ , so we can write

$$a^2 + a^2 = 1$$

$$\cancel{2} \cancel{a^2} = \frac{1}{2}$$

$$\sqrt{a^2} = \sqrt{\frac{1}{2}}$$

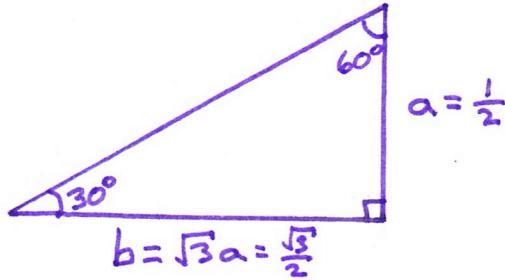


$\Rightarrow a = \frac{1}{\sqrt{2}}$  but we can't leave a root in the denominator

$$a = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

so: 
$$a = b = \frac{\sqrt{2}}{2}$$

for angles that are multiples of  $30^\circ$  &  $60^\circ$



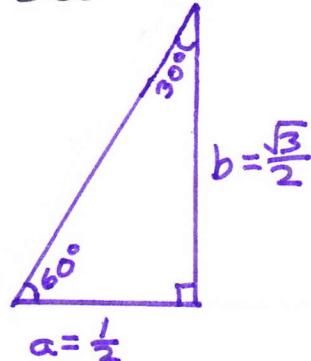
We can find  $a$  &  $b$  because we  
know the hypotenuse = 1  
and we know in a  $30^\circ-60^\circ-90^\circ$   
triangle, the hypotenuse =  $2a$ .

$$\cancel{2}a = \frac{1}{2}$$

$$a = \frac{1}{2}$$

Once we know  $a$ ,  $b = \sqrt{3}a = \sqrt{3}\left(\frac{1}{2}\right)$

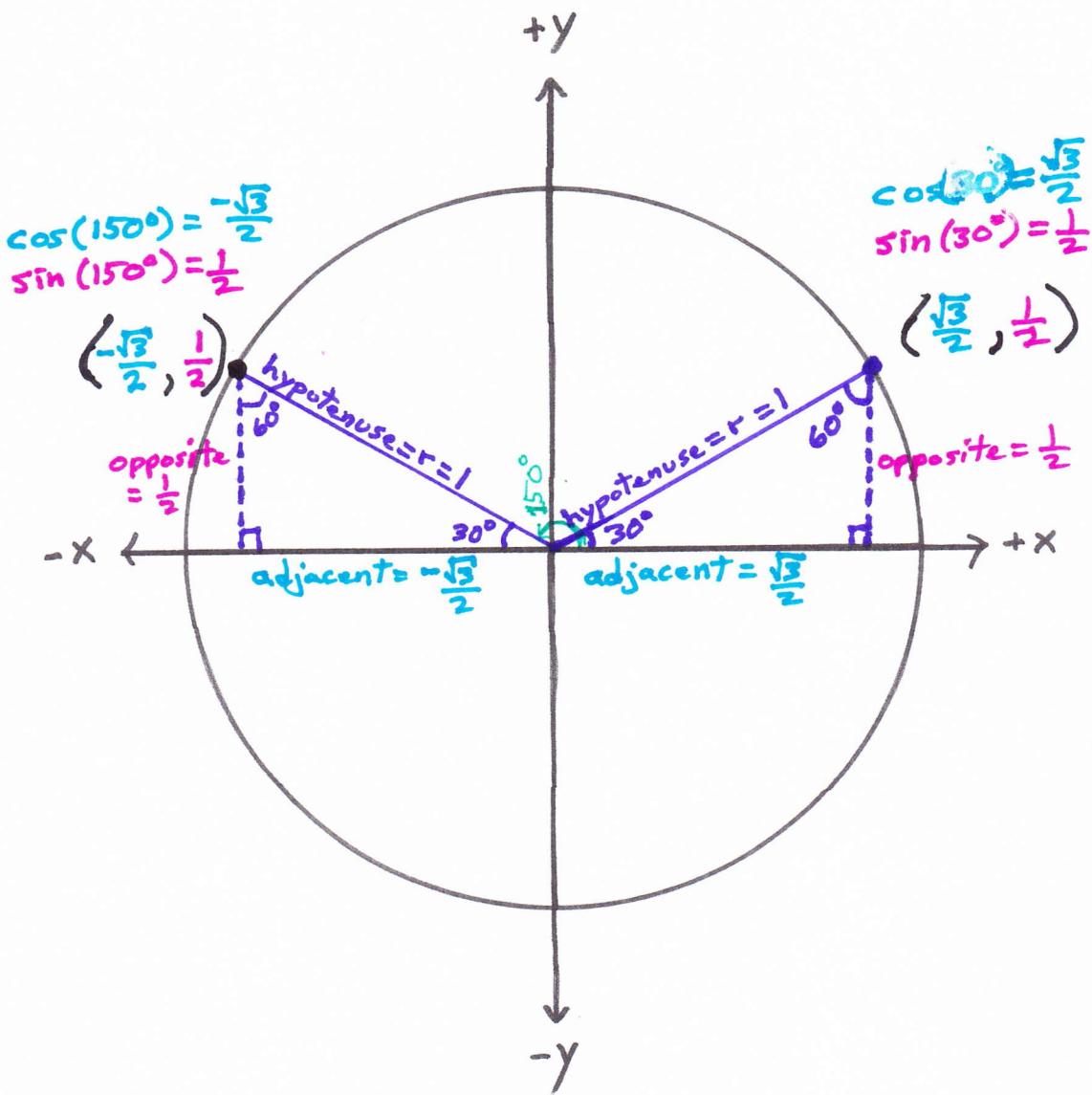
$$\text{so } b = \frac{\sqrt{3}}{2}$$



because  $a$  is the side opposite  
to the  $30^\circ$  angle and  $b$  is the  
side opposite to the  $60^\circ$  angle,

the same calculations apply  
because it is still a  $30^\circ-60^\circ-90^\circ$   
triangle.

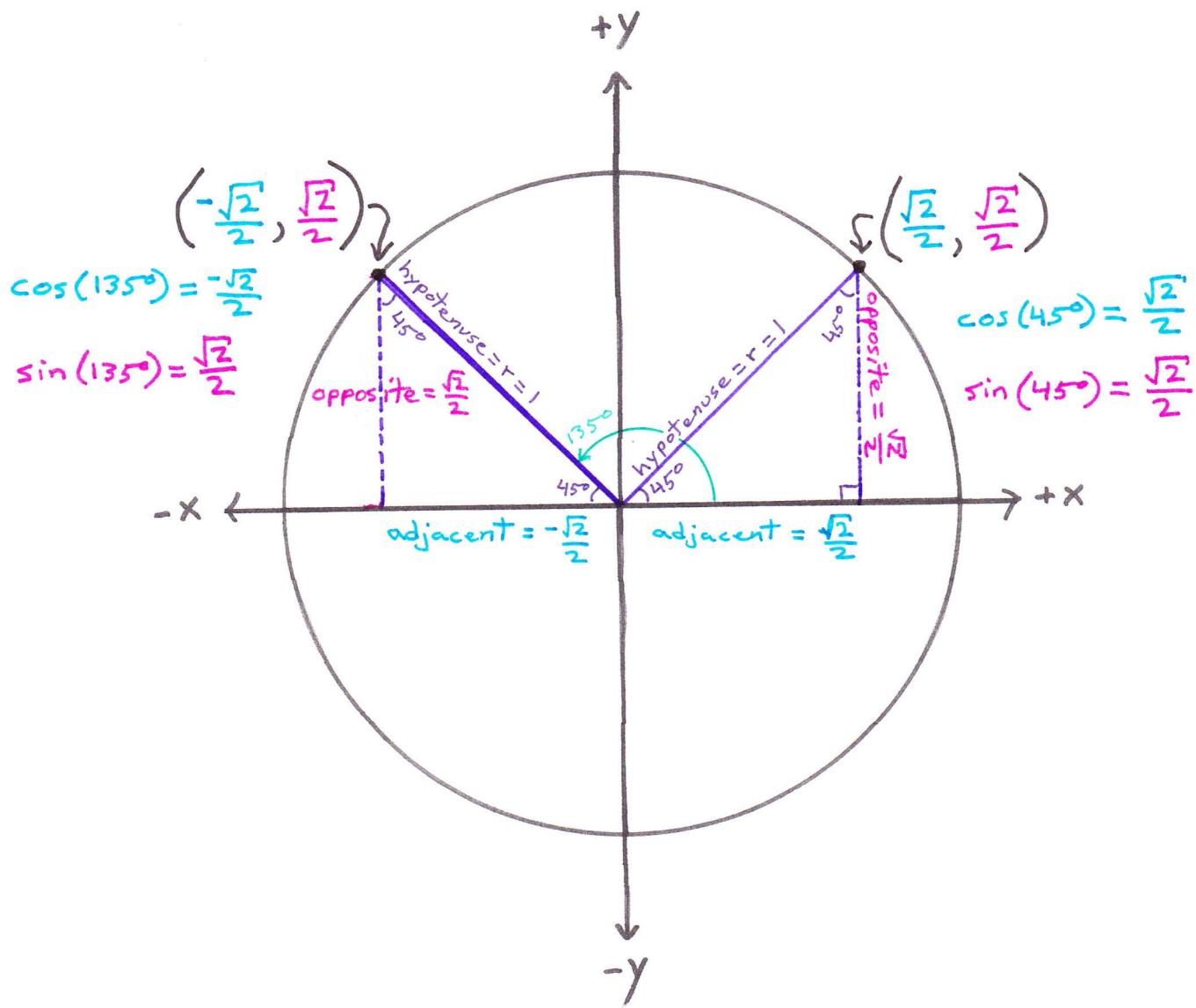
Unit Circle: 1<sup>st</sup> Quadrant,  $30^\circ$ , & 2<sup>nd</sup> Quadrant,  $150^\circ$   
 (+x, +y) (-x, +y)



Did you notice that you can also use symmetry  
 to get the coordinates in quadrant 2 from  
 those in quadrant 1?

Tip: Make sure you measure the angles from the x-axis.

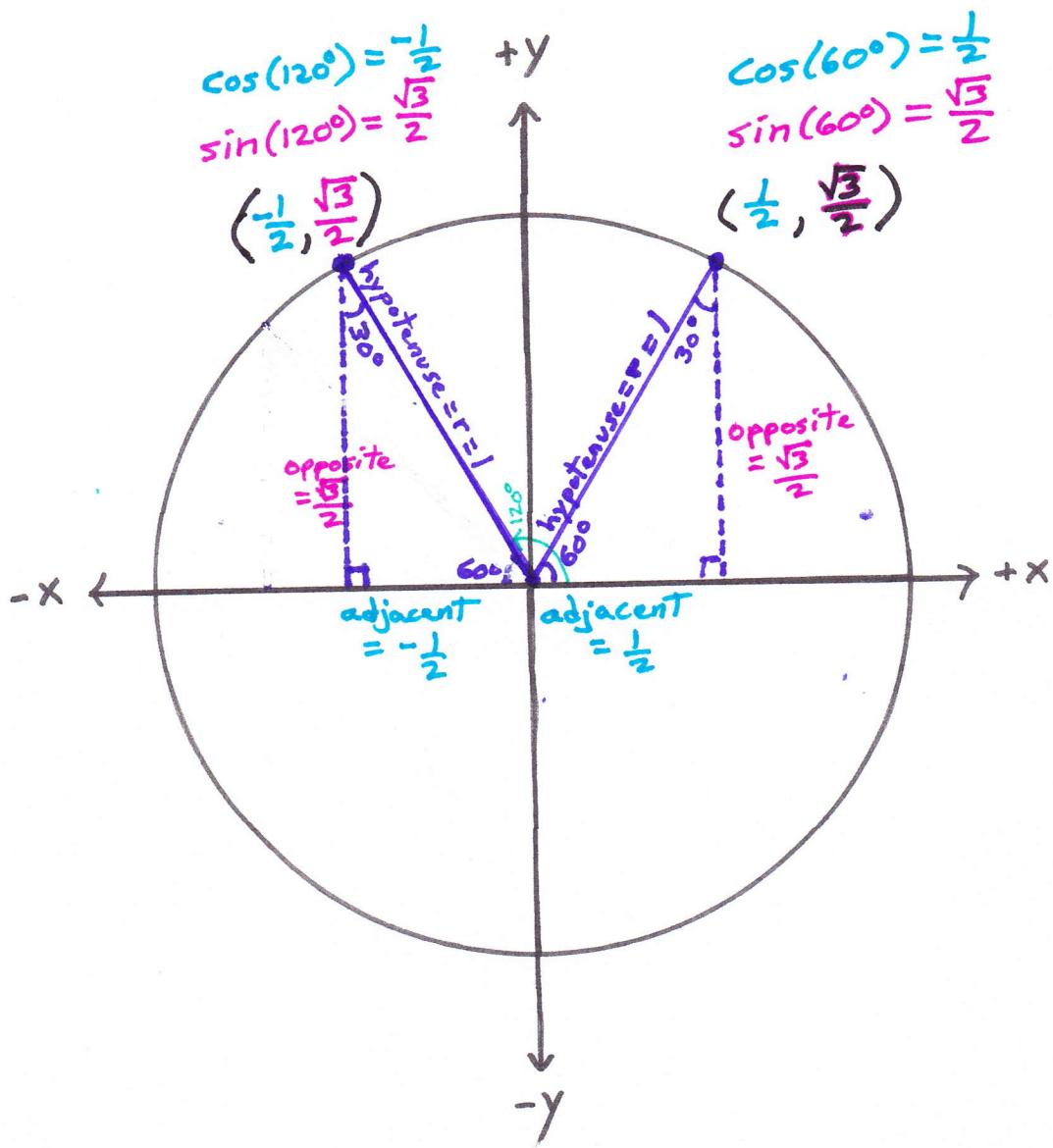
Unit Circle: 1<sup>st</sup> Quadrant,  $45^\circ$ , & 2<sup>nd</sup> Quadrant,  $135^\circ$



Unit Circle: 1<sup>st</sup> Quadrant, 60° & 2<sup>nd</sup> Quadrant, 120°

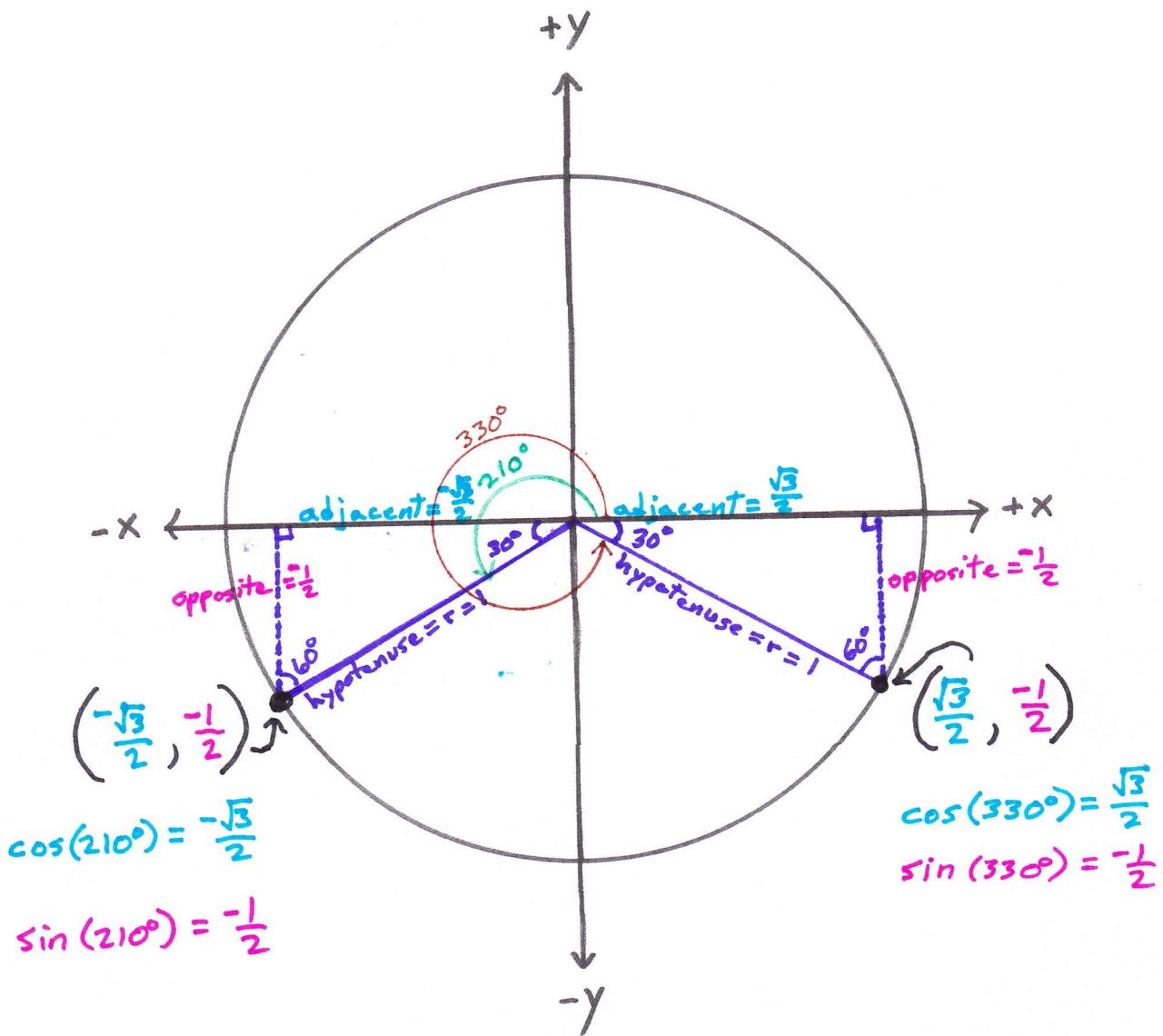
(+x, +y)

(-x, +y)

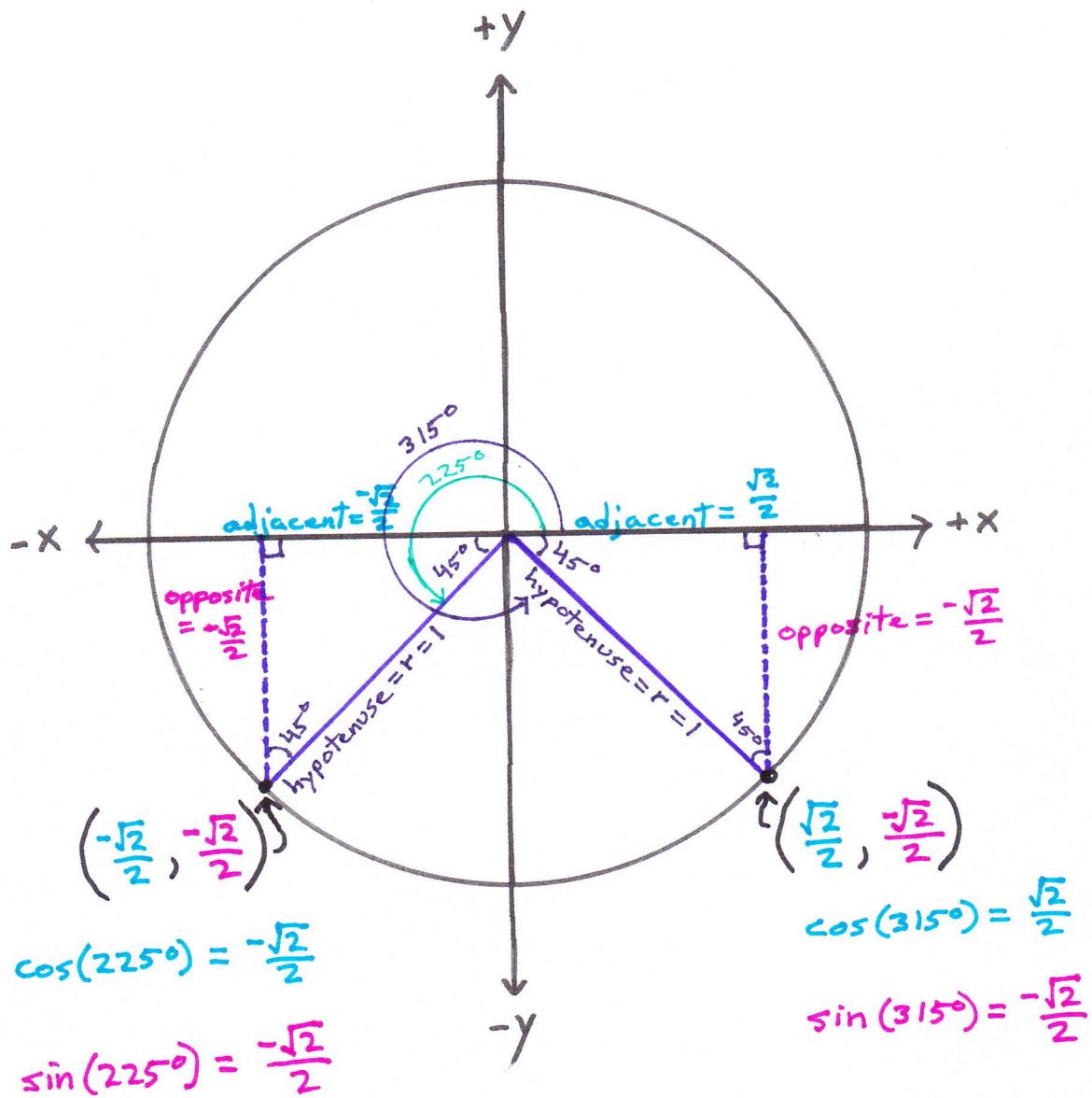


Tip: Use symmetry to cut your work in half.

Unit Circle: 3<sup>rd</sup> Quadrant,  $210^\circ$ , & 4<sup>th</sup> Quadrant,  $330^\circ$   
 (-x, -y) (+x, -y)



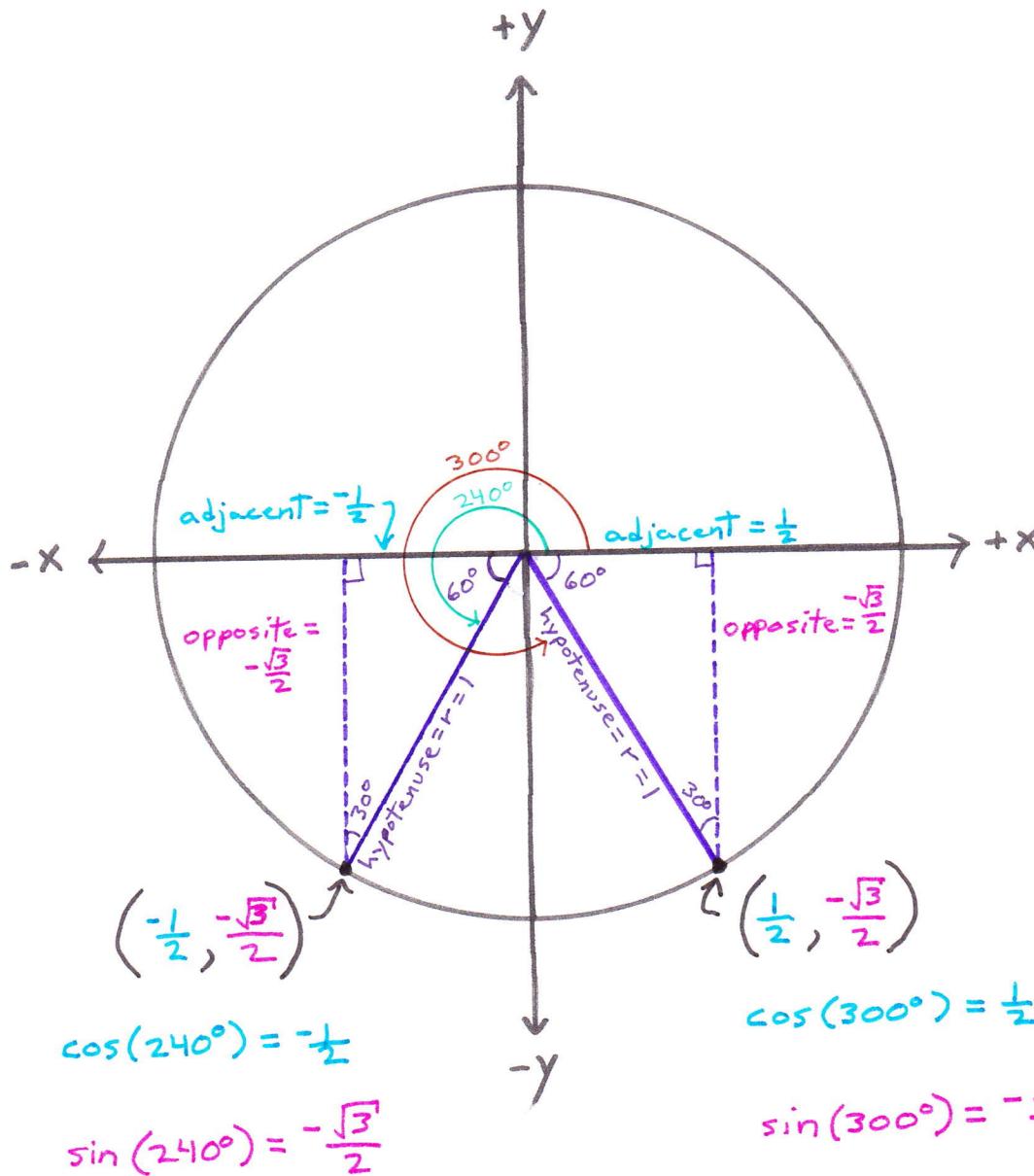
Unit Circle: 3<sup>rd</sup> Quadrant,  $225^\circ$ , & 4<sup>th</sup> Quadrant,  $315^\circ$   
 (-x, -y) (+x, -y)



Unit Circle: 3<sup>rd</sup> Quadrant, 240°, & 4<sup>th</sup> Quadrant, 300°

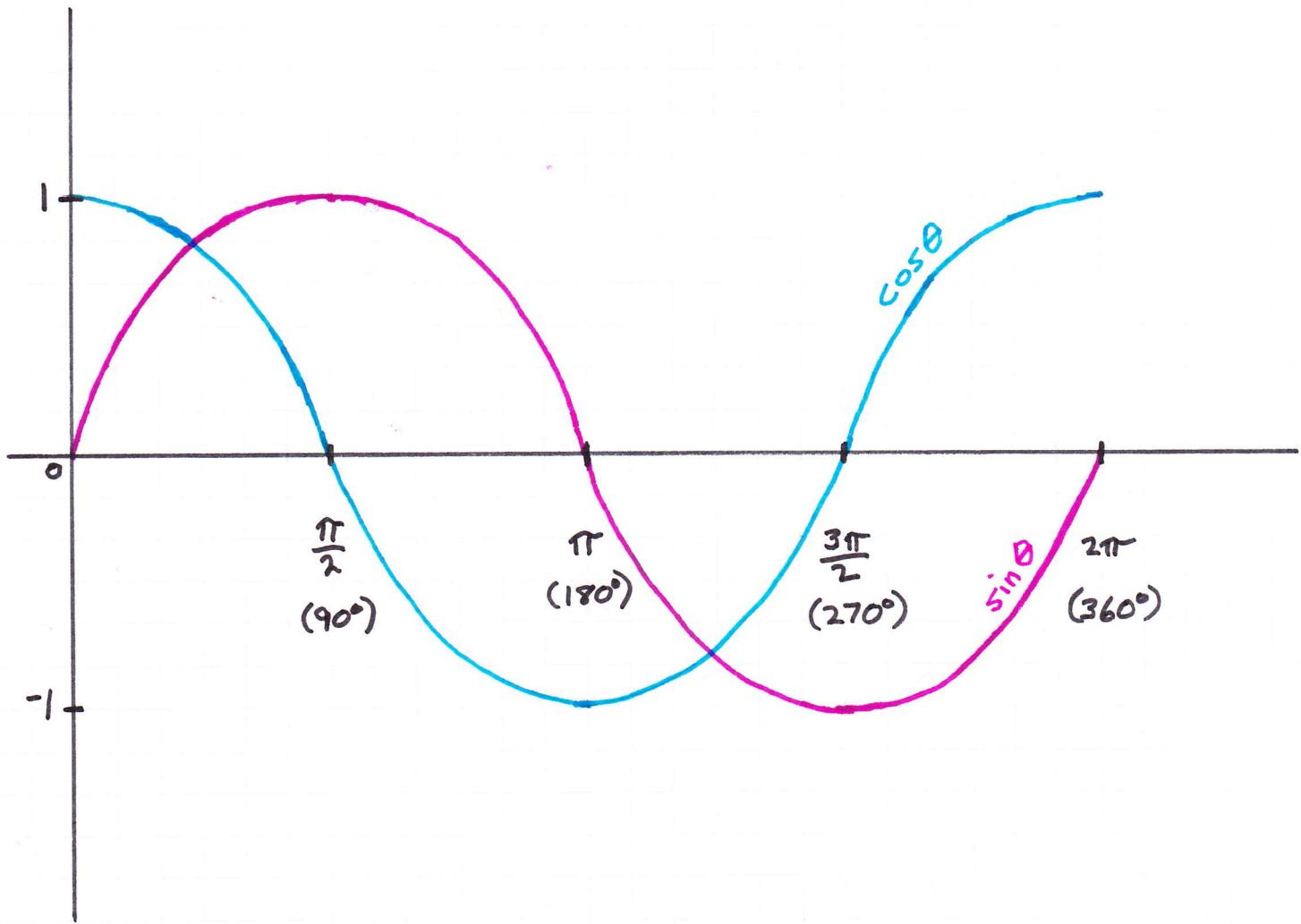
(-x, -y)

(+x, -y)



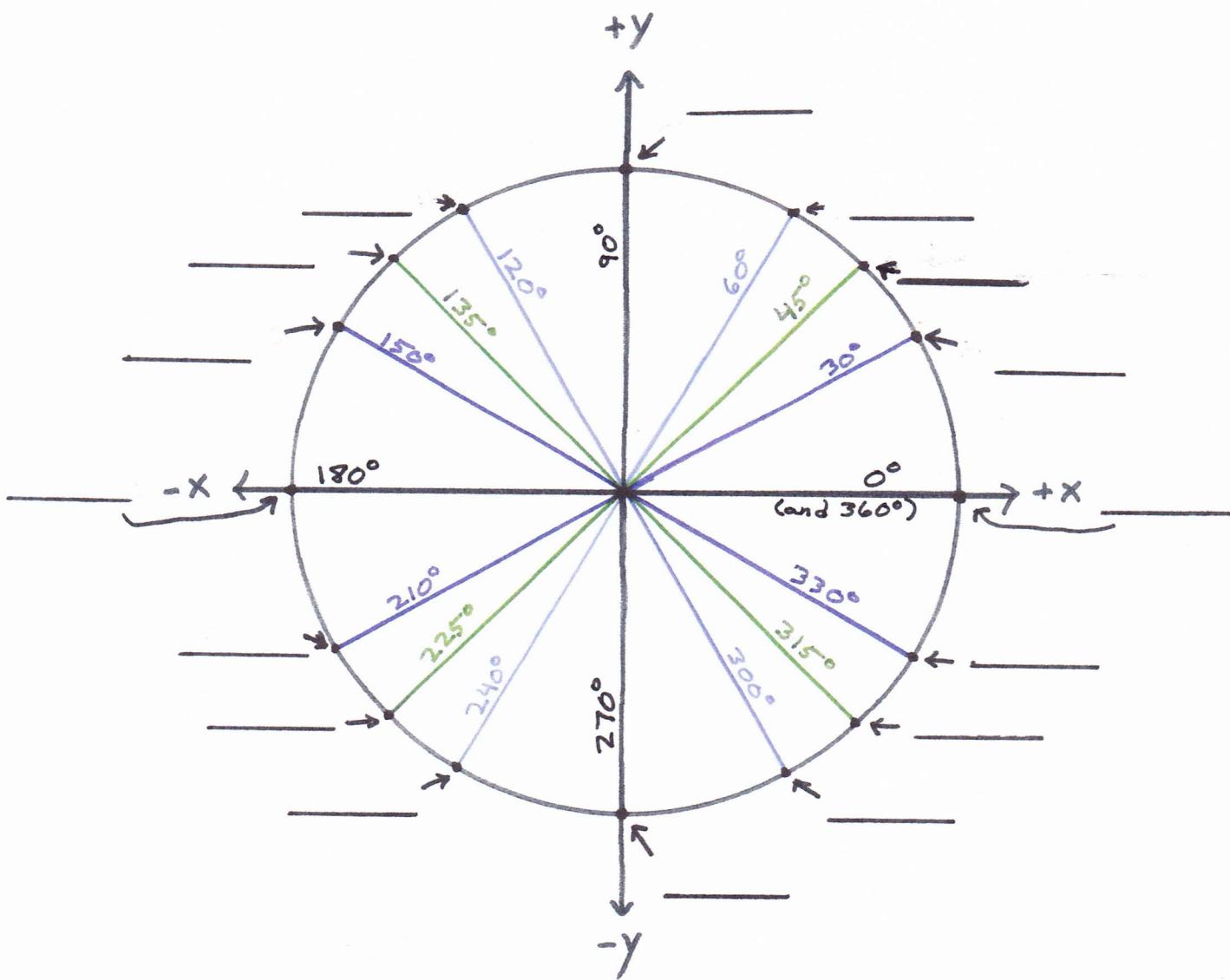
# The Unit Circle

"Unrolled"



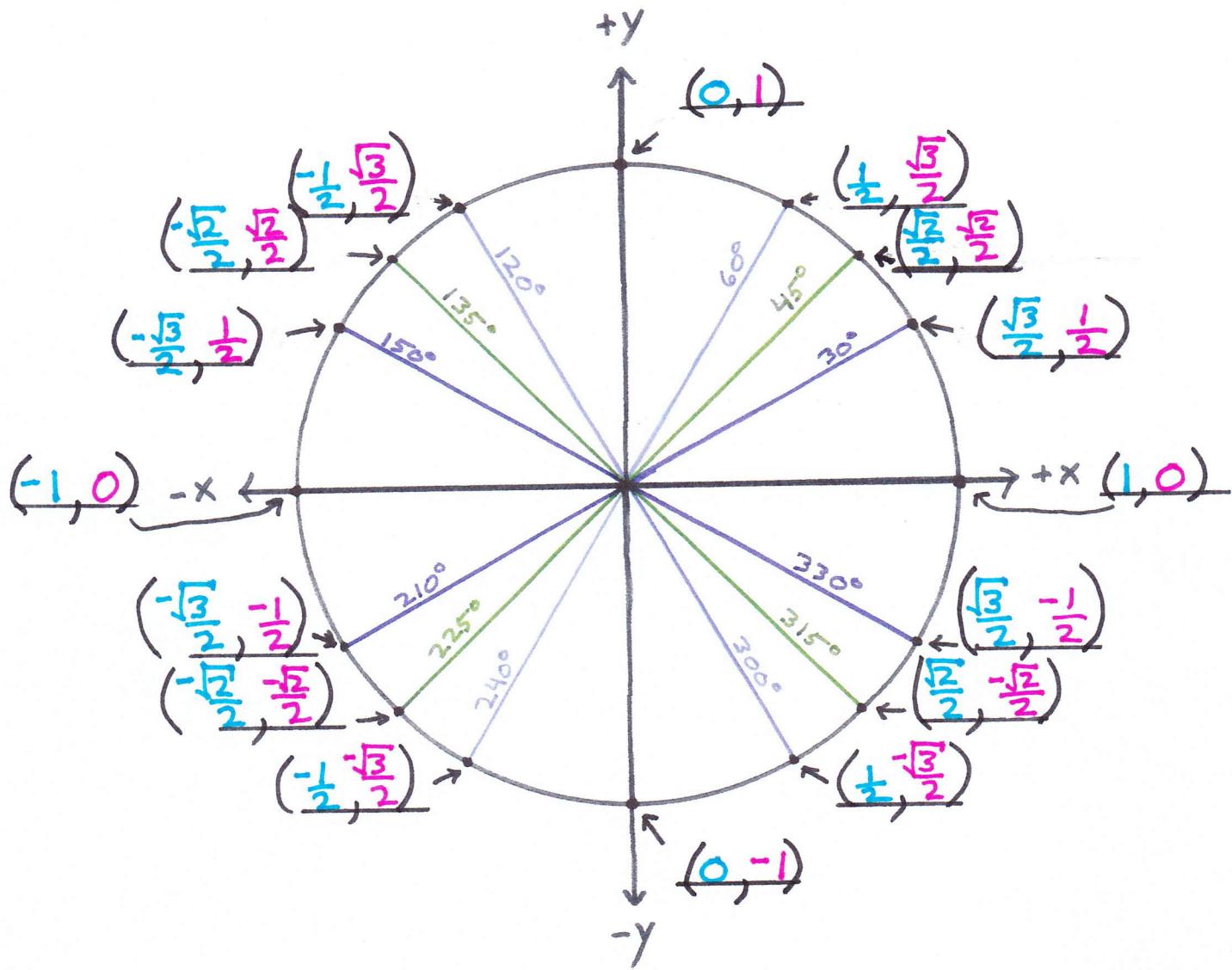
# The Unit Circle: Degrees

Fill in the missing information.  
(ordered pairs)



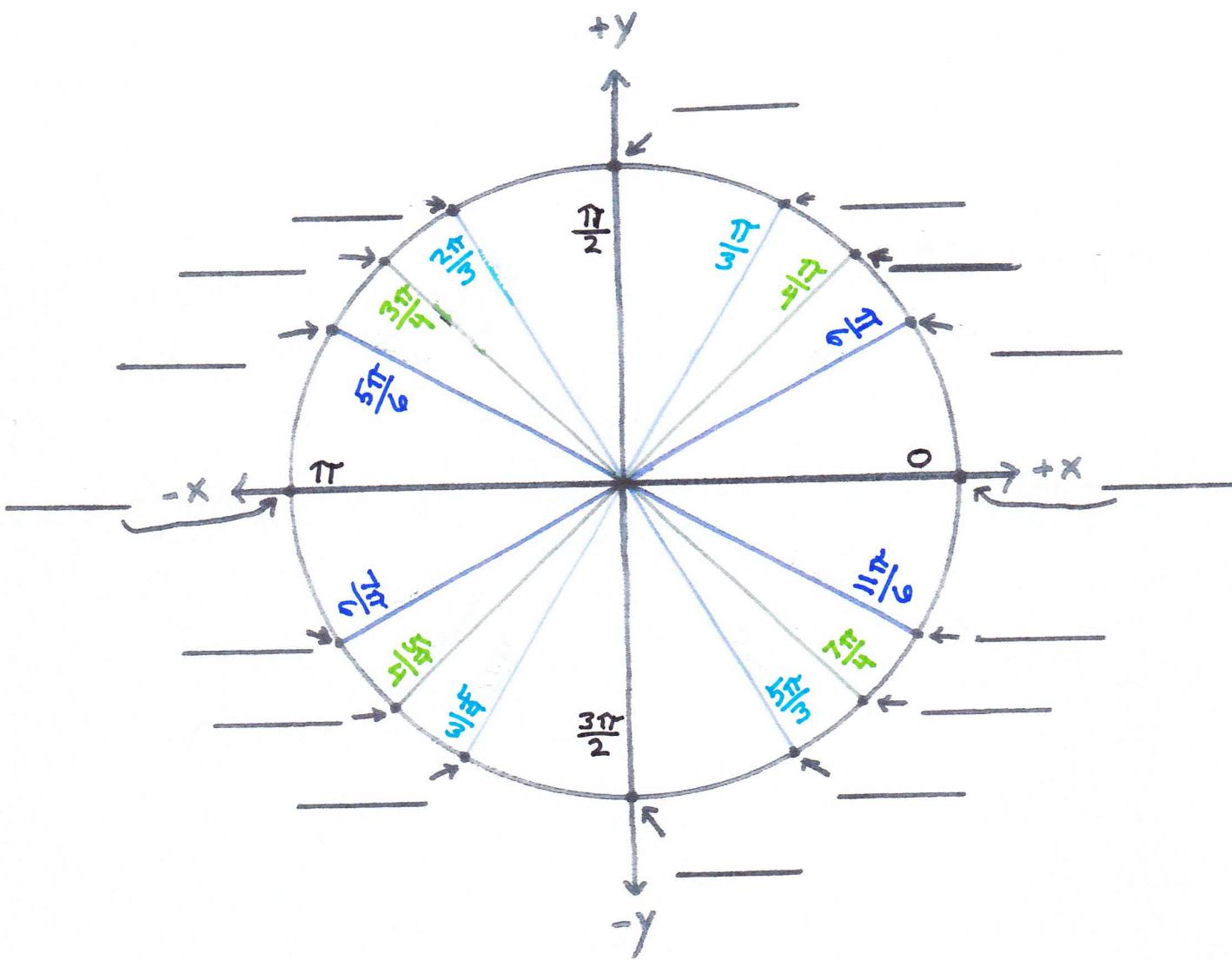
The Unit Circle  
Degrees  
Fill in the missing information.

KEY



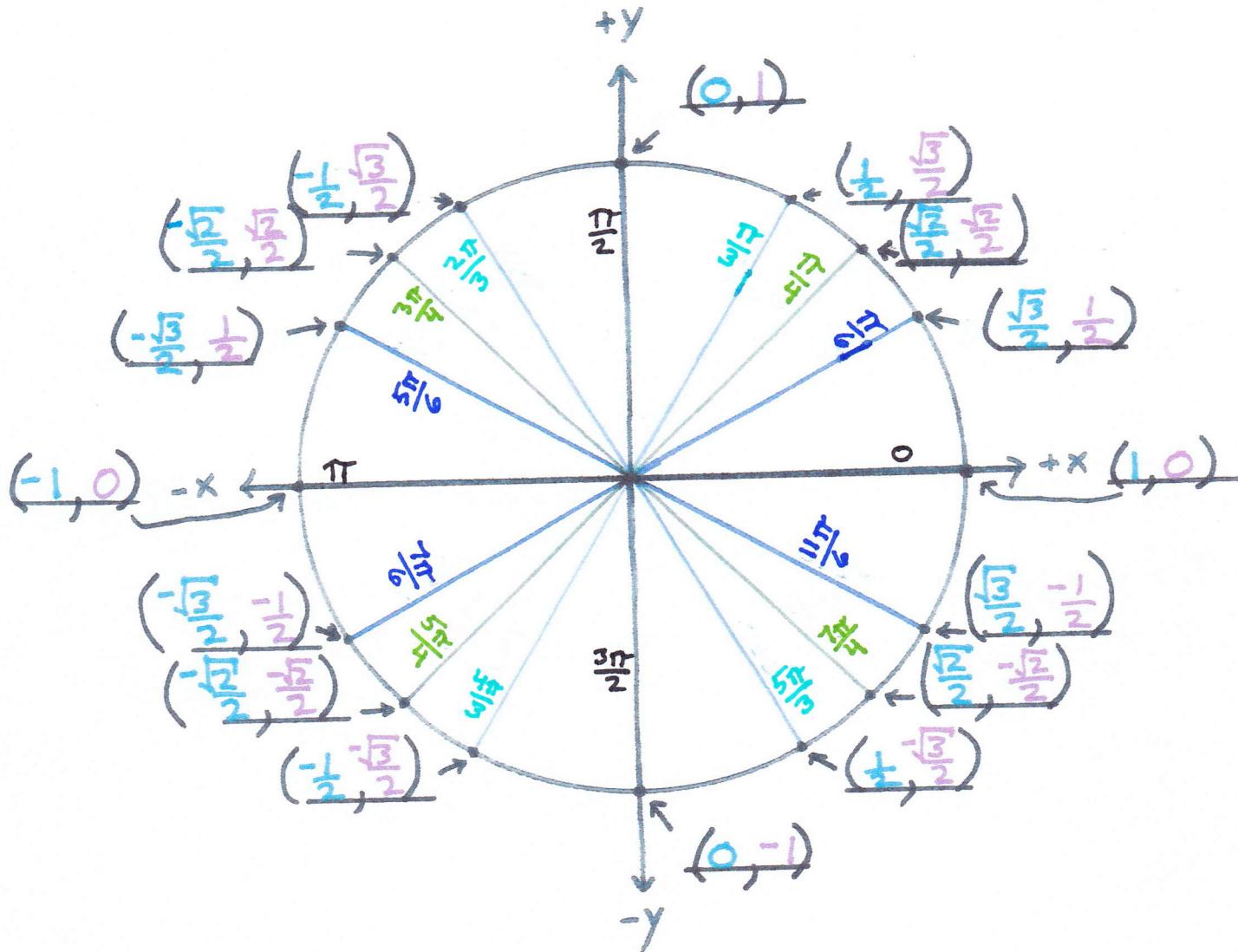
# The Unit Circle: Radians

Fill in the missing information.  
(ordered pairs)



**The Unit Circle**  
**Radians**  
 Fill in the missing information.

KEY



# Convert From Degrees to Radians

$$0^\circ = 0 \text{ radians}$$

$$30^\circ = 30^\circ \cdot \frac{\pi}{360^\circ} = \frac{1}{12}\pi \text{ or } \frac{\pi}{6}$$

$$45^\circ = 45^\circ \cdot \frac{\pi}{360^\circ} = \frac{1}{8}\pi \text{ or } \frac{\pi}{4}$$

$$60^\circ =$$

$$90^\circ =$$

$$120^\circ =$$

$$135^\circ =$$

$$150^\circ =$$

$$180^\circ =$$

$$210^\circ =$$

$$225^\circ =$$

$$240^\circ =$$

$$270^\circ =$$

$$300^\circ =$$

$$315^\circ =$$

$$330^\circ =$$

$$360^\circ =$$

Convert from  
Degrees to Radians  
**KEY**

$$0^\circ = 0 \text{ radians}$$

$$30^\circ = 30^\circ \cdot \frac{\frac{2\pi}{3}}{360^\circ} = \frac{1}{6}\pi \text{ or } \frac{\pi}{6}$$

$$45^\circ = 45^\circ \cdot \frac{\frac{2\pi}{3}}{360^\circ} = \frac{1}{4}\pi \text{ or } \frac{\pi}{4}$$

$$60^\circ = \frac{1}{3}\pi \text{ or } \frac{\pi}{3}$$

$$90^\circ = \frac{1}{2}\pi \text{ or } \frac{\pi}{2}$$

$$120^\circ = \frac{2}{3}\pi \text{ or } \frac{2\pi}{3}$$

$$135^\circ = \frac{3}{4}\pi \text{ or } \frac{3\pi}{4}$$

$$150^\circ = \frac{5}{6}\pi \text{ or } \frac{5\pi}{6}$$

$$180^\circ = \pi \quad \leftarrow \text{ just } \pi \text{ ☺}$$

$$210^\circ = \frac{7}{6}\pi \text{ or } \frac{7\pi}{6}$$

$$225^\circ = \frac{5}{4}\pi \text{ or } \frac{5\pi}{4}$$

$$240^\circ = \frac{4}{3}\pi \text{ or } \frac{4\pi}{3}$$

$$270^\circ = \frac{3}{2}\pi \text{ or } \frac{3\pi}{2}$$

$$300^\circ = \frac{5}{3}\pi \text{ or } \frac{5\pi}{3}$$

$$315^\circ = \frac{7}{4}\pi \text{ or } \frac{7\pi}{4}$$

$$330^\circ = \frac{11}{6}\pi \text{ or } \frac{11\pi}{6}$$

$$360^\circ = 2\pi \quad \leftarrow \text{ full circle ☺}$$

# The Unit Circle

## Degrees

Fill in the following using your unit circle worksheet as a reference.

$$\sin(0^\circ) = \underline{\hspace{2cm}} \quad \cos(0^\circ) = \underline{\hspace{2cm}} \quad \tan(0^\circ) = \underline{\hspace{2cm}}$$

$$\sin(90^\circ) = \underline{\hspace{2cm}} \quad \cos(90^\circ) = \underline{\hspace{2cm}} \quad \tan(90^\circ) = \underline{\hspace{2cm}}$$

$$\sin(180^\circ) = \underline{\hspace{2cm}} \quad \cos(180^\circ) = \underline{\hspace{2cm}} \quad \tan(180^\circ) = \underline{\hspace{2cm}}$$

$$\sin(270^\circ) = \underline{\hspace{2cm}} \quad \cos(270^\circ) = \underline{\hspace{2cm}} \quad \tan(270^\circ) = \underline{\hspace{2cm}}$$

.....

$$\sin(30^\circ) = \underline{\hspace{2cm}} \quad \cos(30^\circ) = \underline{\hspace{2cm}} \quad \tan(30^\circ) = \underline{\hspace{2cm}}$$

$$\sin(150^\circ) = \underline{\hspace{2cm}} \quad \cos(150^\circ) = \underline{\hspace{2cm}} \quad \tan(150^\circ) = \underline{\hspace{2cm}}$$

$$\sin(210^\circ) = \underline{\hspace{2cm}} \quad \cos(210^\circ) = \underline{\hspace{2cm}} \quad \tan(210^\circ) = \underline{\hspace{2cm}}$$

$$\sin(330^\circ) = \underline{\hspace{2cm}} \quad \cos(330^\circ) = \underline{\hspace{2cm}} \quad \tan(330^\circ) = \underline{\hspace{2cm}}$$

.....

$$\sin(45^\circ) = \underline{\hspace{2cm}} \quad \cos(45^\circ) = \underline{\hspace{2cm}} \quad \tan(45^\circ) = \underline{\hspace{2cm}}$$

$$\sin(135^\circ) = \underline{\hspace{2cm}} \quad \cos(135^\circ) = \underline{\hspace{2cm}} \quad \tan(135^\circ) = \underline{\hspace{2cm}}$$

$$\sin(225^\circ) = \underline{\hspace{2cm}} \quad \cos(225^\circ) = \underline{\hspace{2cm}} \quad \tan(225^\circ) = \underline{\hspace{2cm}}$$

$$\sin(315^\circ) = \underline{\hspace{2cm}} \quad \cos(315^\circ) = \underline{\hspace{2cm}} \quad \tan(315^\circ) = \underline{\hspace{2cm}}$$

.....

$$\sin(60^\circ) = \underline{\hspace{2cm}} \quad \cos(60^\circ) = \underline{\hspace{2cm}} \quad \tan(60^\circ) = \underline{\hspace{2cm}}$$

$$\sin(120^\circ) = \underline{\hspace{2cm}} \quad \cos(120^\circ) = \underline{\hspace{2cm}} \quad \tan(120^\circ) = \underline{\hspace{2cm}}$$

$$\sin(240^\circ) = \underline{\hspace{2cm}} \quad \cos(240^\circ) = \underline{\hspace{2cm}} \quad \tan(240^\circ) = \underline{\hspace{2cm}}$$

$$\sin(300^\circ) = \underline{\hspace{2cm}} \quad \cos(300^\circ) = \underline{\hspace{2cm}} \quad \tan(300^\circ) = \underline{\hspace{2cm}}$$

What patterns do you see?

# The Unit Circle: Degrees **KEY**

Fill in the following using your unit circle worksheet as a reference.

$$\sin(0^\circ) = \underline{\quad 0 \quad}$$

$$\cos(0^\circ) = \underline{\quad 1 \quad}$$

$$\tan(0^\circ) = \underline{\frac{0}{1}} = 0$$

$$\sin(90^\circ) = \underline{\quad 1 \quad}$$

$$\cos(90^\circ) = \underline{\quad 0 \quad}$$

$$\tan(90^\circ) = \underline{\frac{1}{0}} = \text{undefined}$$

$$\sin(180^\circ) = \underline{\quad 0 \quad}$$

$$\cos(180^\circ) = \underline{\quad -1 \quad}$$

$$\tan(180^\circ) = \underline{\frac{0}{-1}} = 0$$

$$\sin(270^\circ) = \underline{\quad -1 \quad}$$

$$\cos(270^\circ) = \underline{\quad 0 \quad}$$

$$\tan(270^\circ) = \underline{\frac{-1}{0}} = \text{undefined}$$

$$\sin(30^\circ) = \underline{\quad \pm \frac{1}{2} \quad}$$

$$\cos(30^\circ) = \underline{\quad \frac{\sqrt{3}}{2} \quad}$$

$$\tan(30^\circ) = \underline{\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sin(150^\circ) = \underline{\quad \frac{1}{2} \quad}$$

$$\cos(150^\circ) = \underline{\quad -\frac{\sqrt{3}}{2} \quad}$$

$$\tan(150^\circ) = \underline{\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}} = \frac{1}{2} \cdot \frac{-2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\sin(210^\circ) = \underline{\quad -\frac{1}{2} \quad}$$

$$\cos(210^\circ) = \underline{\quad -\frac{\sqrt{3}}{2} \quad}$$

$$\tan(210^\circ) = \underline{\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}} = -\frac{1}{2} \cdot \frac{-2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sin(330^\circ) = \underline{\quad -\frac{1}{2} \quad}$$

$$\cos(330^\circ) = \underline{\quad \frac{\sqrt{3}}{2} \quad}$$

$$\tan(330^\circ) = \underline{\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}} = -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sin(45^\circ) = \underline{\quad \frac{\sqrt{2}}{2} \quad}$$

$$\cos(45^\circ) = \underline{\quad \frac{\sqrt{2}}{2} \quad}$$

$$\tan(45^\circ) = \underline{\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}} = 1$$

$$\sin(135^\circ) = \underline{\quad \frac{\sqrt{2}}{2} \quad}$$

$$\cos(135^\circ) = \underline{\quad -\frac{\sqrt{2}}{2} \quad}$$

$$\tan(135^\circ) = \underline{\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}} = -1$$

$$\sin(225^\circ) = \underline{\quad -\frac{\sqrt{2}}{2} \quad}$$

$$\cos(225^\circ) = \underline{\quad -\frac{\sqrt{2}}{2} \quad}$$

$$\tan(225^\circ) = \underline{\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}} = 1$$

$$\sin(315^\circ) = \underline{\quad -\frac{\sqrt{2}}{2} \quad}$$

$$\cos(315^\circ) = \underline{\quad \frac{\sqrt{2}}{2} \quad}$$

$$\tan(315^\circ) = \underline{\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}} = -1$$

$$\sin(60^\circ) = \underline{\quad \frac{\sqrt{3}}{2} \quad}$$

$$\cos(60^\circ) = \underline{\quad \frac{1}{2} \quad}$$

$$\tan(60^\circ) = \underline{\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\sin(120^\circ) = \underline{\quad \frac{\sqrt{3}}{2} \quad}$$

$$\cos(120^\circ) = \underline{\quad -\frac{1}{2} \quad}$$

$$\tan(120^\circ) = \underline{\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}} = \frac{\sqrt{3}}{2} \cdot \frac{-2}{1} = -\sqrt{3}$$

$$\sin(240^\circ) = \underline{\quad -\frac{\sqrt{3}}{2} \quad}$$

$$\cos(240^\circ) = \underline{\quad -\frac{1}{2} \quad}$$

$$\tan(240^\circ) = \underline{\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}} = -\frac{\sqrt{3}}{2} \cdot \frac{-2}{1} = \sqrt{3}$$

$$\sin(300^\circ) = \underline{\quad -\frac{\sqrt{3}}{2} \quad}$$

$$\cos(300^\circ) = \underline{\quad \frac{1}{2} \quad}$$

$$\tan(300^\circ) = \underline{\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$$

# The Unit Circle

## Radians

Fill in the following using your unit circle

worksheet as a reference. (The order of the given angles is not the same as the "degrees" worksheet.)

$$\sin(0) = \underline{\hspace{2cm}} \quad \cos(0) = \underline{\hspace{2cm}} \quad \tan(0) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{\pi}{6}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{\pi}{6}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{\pi}{6}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{\pi}{3}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{\pi}{3}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{\pi}{3}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{\pi}{2}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{\pi}{2}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{\pi}{2}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{2\pi}{3}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{2\pi}{3}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{2\pi}{3}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{3\pi}{4}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{3\pi}{4}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{3\pi}{4}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{5\pi}{6}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{5\pi}{6}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{5\pi}{6}\right) = \underline{\hspace{2cm}}$$

$$\sin(\pi) = \underline{\hspace{2cm}} \quad \cos(\pi) = \underline{\hspace{2cm}} \quad \tan(\pi) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{7\pi}{6}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{7\pi}{6}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{7\pi}{6}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{5\pi}{4}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{5\pi}{4}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{5\pi}{4}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{4\pi}{3}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{4\pi}{3}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{4\pi}{3}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{3\pi}{2}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{3\pi}{2}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{3\pi}{2}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{5\pi}{3}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{5\pi}{3}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{5\pi}{3}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{7\pi}{4}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{7\pi}{4}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{7\pi}{4}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{11\pi}{6}\right) = \underline{\hspace{2cm}} \quad \cos\left(\frac{11\pi}{6}\right) = \underline{\hspace{2cm}} \quad \tan\left(\frac{11\pi}{6}\right) = \underline{\hspace{2cm}}$$

What patterns do you see?

# The Unit Circle

Radians

**KEY**

Fill in the following using your unit circle worksheet as a reference. (The order of the given angles is not the same as the "degrees" worksheet.)

$$\sin(0) = \underline{\underline{0}}$$

$$\cos(0) = \underline{\underline{1}}$$

$$\tan(0) = \frac{\underline{\underline{0}}}{\underline{\underline{1}}} = 0$$

$$\sin(\frac{\pi}{6}) = \underline{\underline{\frac{1}{2}}}$$

$$\cos(\frac{\pi}{6}) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$\tan(\frac{\pi}{6}) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sin(\frac{\pi}{4}) = \underline{\underline{\frac{\sqrt{2}}{2}}}$$

$$\cos(\frac{\pi}{4}) = \underline{\underline{\frac{\sqrt{2}}{2}}}$$

$$\tan(\frac{\pi}{4}) = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}} = 1$$

$$\sin(\frac{\pi}{3}) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$\cos(\frac{\pi}{3}) = \underline{\underline{\frac{1}{2}}}$$

$$\tan(\frac{\pi}{3}) = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

$$\sin(\frac{\pi}{2}) = \underline{\underline{-1}}$$

$$\cos(\frac{\pi}{2}) = \underline{\underline{0}}$$

$$\tan(\frac{\pi}{2}) = \frac{0}{-1} \rightarrow \text{undefined}$$

$$\sin(\frac{2\pi}{3}) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$\cos(\frac{2\pi}{3}) = \underline{\underline{-\frac{1}{2}}}$$

$$\tan(\frac{2\pi}{3}) = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{-2}{1} = -\sqrt{3}$$

$$\sin(\frac{3\pi}{4}) = \underline{\underline{\frac{\sqrt{2}}{2}}}$$

$$\cos(\frac{3\pi}{4}) = \underline{\underline{-\frac{\sqrt{2}}{2}}}$$

$$\tan(\frac{3\pi}{4}) = \frac{\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = -1$$

$$\sin(\frac{5\pi}{6}) = \underline{\underline{\frac{1}{2}}}$$

$$\cos(\frac{5\pi}{6}) = \underline{\underline{-\frac{\sqrt{3}}{2}}}$$

$$\tan(\frac{5\pi}{6}) = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{-2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\sin(\pi) = \underline{\underline{0}}$$

$$\cos(\pi) = \underline{\underline{-1}}$$

$$\tan(\pi) = \frac{0}{-1} = 0$$

$$\sin(\frac{7\pi}{6}) = \underline{\underline{-\frac{1}{2}}}$$

$$\cos(\frac{7\pi}{6}) = \underline{\underline{-\frac{\sqrt{3}}{2}}}$$

$$\tan(\frac{7\pi}{6}) = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{-2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sin(\frac{5\pi}{4}) = \underline{\underline{-\frac{\sqrt{2}}{2}}}$$

$$\cos(\frac{5\pi}{4}) = \underline{\underline{-\frac{\sqrt{2}}{2}}}$$

$$\tan(\frac{5\pi}{4}) = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$$

$$\sin(\frac{4\pi}{3}) = \underline{\underline{-\frac{\sqrt{3}}{2}}}$$

$$\cos(\frac{4\pi}{3}) = \underline{\underline{-\frac{1}{2}}}$$

$$\tan(\frac{4\pi}{3}) = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{-2}{1} = \sqrt{3}$$

$$\sin(\frac{3\pi}{2}) = \underline{\underline{-1}}$$

$$\cos(\frac{3\pi}{2}) = \underline{\underline{0}}$$

$$\tan(\frac{3\pi}{2}) = \frac{-1}{0} \rightarrow \text{undefined}$$

$$\sin(\frac{5\pi}{3}) = \underline{\underline{-\frac{\sqrt{3}}{2}}}$$

$$\cos(\frac{5\pi}{3}) = \underline{\underline{\frac{1}{2}}}$$

$$\tan(\frac{5\pi}{3}) = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$$

$$\sin(\frac{7\pi}{4}) = \underline{\underline{-\frac{\sqrt{2}}{2}}}$$

$$\cos(\frac{7\pi}{4}) = \underline{\underline{\frac{\sqrt{2}}{2}}}$$

$$\tan(\frac{7\pi}{4}) = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$$

$$\sin(\frac{11\pi}{6}) = \underline{\underline{-\frac{1}{2}}}$$

$$\cos(\frac{11\pi}{6}) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

$$\tan(\frac{11\pi}{6}) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

What patterns do you see?

# Inverse Trig Functions

Secant:  $\sec \theta = \cos^{-1} \theta = \frac{1}{\cos \theta}$

Cosecant:  $\csc \theta = \sin^{-1} \theta = \frac{1}{\sin \theta}$

Cotangent  $\cot \theta = \tan^{-1} \theta = \frac{1}{\tan \theta}$

Find the following:

$$1. \sec\left(\frac{\pi}{3}\right)$$

$$2. \csc\left(\frac{5\pi}{6}\right)$$

$$3. \cot\left(\frac{5\pi}{2}\right)$$

$$4. \sec\left(\frac{11\pi}{6}\right)$$

$$5. \csc\left(\frac{2\pi}{3}\right)$$

$$6. \cot\left(\frac{4\pi}{3}\right)$$

$$7. \sec(210^\circ)$$

$$8. \csc(120^\circ)$$

$$9. \cot(330^\circ)$$

$$10. \sec(30^\circ)$$

$$11. \csc(135^\circ)$$

$$12. \cot(270^\circ)$$

# KEY

## Inverse Trig Functions

Secant:  $\sec \theta = \cos^{-1} \theta = \frac{1}{\cos \theta}$

Cosecant:  $\csc \theta = \sin^{-1} \theta = \frac{1}{\sin \theta}$

Cotangent  $\cot \theta = \tan^{-1} \theta = \frac{1}{\tan \theta}$

Find the following:

1.  $\sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 1 \cdot \frac{2}{1} = \boxed{2}$

2.  $\csc\left(\frac{5\pi}{6}\right) = \frac{1}{\sin\left(\frac{5\pi}{6}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 1 \cdot \left(\frac{2}{1}\right) = \boxed{2}$

3.  $\cot\left(\frac{5\pi}{6}\right) = \frac{1}{\tan\left(\frac{5\pi}{6}\right)} = \frac{1}{\left(-\frac{\sqrt{3}}{3}\right)} = 1 \cdot \left(-\frac{\sqrt{3}}{3}\right) = \boxed{-\frac{\sqrt{3}}{3}}$

4.  $\sec\left(\frac{11\pi}{6}\right) = \frac{1}{\cos\left(\frac{11\pi}{6}\right)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1 \cdot \left(\frac{2}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

5.  $\csc\left(\frac{2\pi}{3}\right) = \frac{1}{\sin\left(\frac{2\pi}{3}\right)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1 \cdot \left(\frac{2}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

6.  $\cot\left(\frac{4\pi}{3}\right) = \frac{1}{\tan\left(\frac{4\pi}{3}\right)} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

7.  $\sec(210^\circ) = \frac{1}{\cos(210^\circ)} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = 1 \cdot \left(-\frac{2}{\sqrt{3}}\right) = \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

8.  $\csc(120^\circ) = \frac{1}{\sin(120^\circ)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

9.  $\cot(330^\circ) = \frac{1}{\tan(330^\circ)} = \frac{1}{\left(-\frac{1}{\sqrt{3}}\right)} = 1 \cdot -\frac{\sqrt{3}}{1} = \boxed{-\sqrt{3}}$

10.  $\sec(30^\circ) = \frac{1}{\cos(30^\circ)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

11.  $\csc(135^\circ) = \frac{1}{\sin(135^\circ)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1 \cdot \left(\frac{2}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{2\sqrt{2}}{2}} = \boxed{\sqrt{2}}$

12.  $\cot(270^\circ) = \frac{1}{\tan(270^\circ)} = \frac{1}{\left(\frac{1}{0}\right)} = 1 \cdot \frac{0}{1} = 1 \cdot 0 = \boxed{0}$

or  $\cot(270^\circ) = \frac{\cos\theta}{\sin\theta} = \frac{0}{-1} = 0$